Clustering in multivariate extremes

A case study of Dutch hydrological extremes

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Joint work with Bas Matsuura[2] and Ioannis Papastathopoulos[1]

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Motivation and problem

- The Netherlands is vulnerable to compound hydrological extremes (Rhine, Meuse, North Sea).
- Extremes persist and cluster across days; duration matters for risk and impact.
- Need joint modelling of low and high discharges with coherent dependence.

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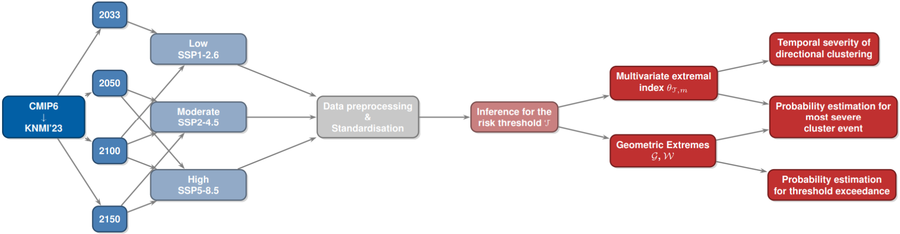
Modelling workflow



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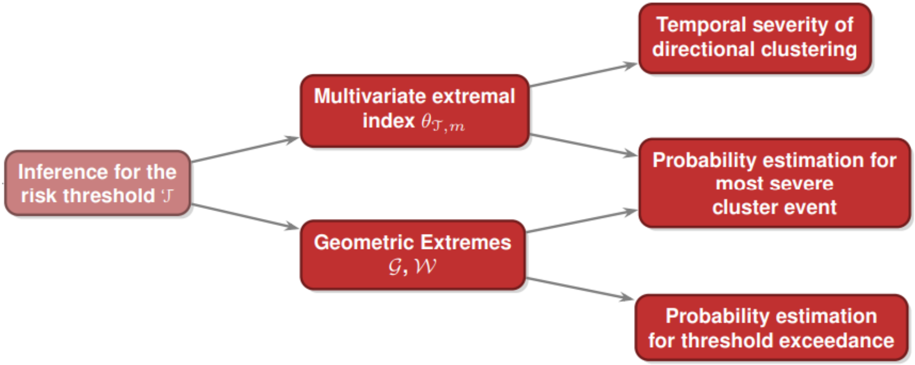

Extremal index (univariate): Block maxima, POT, and Runs estimator

- Consider a sequence $\{X_i\}_{i=1}^{n}$ and define the block maximum $M_n = \max(X_1, \ldots, X_n)$.

- IID case (in the MDA of GEV):

$$\mathbb{P}\left(\frac{M_n - b_n}{a_n} \leq x\right) \longrightarrow G(x;\xi) = \exp\left\{-\left(1+\xi x\right)^{-1/\xi}\right\}, \quad 1+\xi x > 0.$$

- With temporal dependence, *i.e.* stationary sequences, and extremal index $\theta \in (0,1]$:

$$P\left(\frac{M_n - b_n}{a_n} \leq x\right) \longrightarrow G_{\xi}(x)^{\,\theta}.$$

- Consequences: adjust POT inference for dependence via declustering or θ; mean cluster size $\approx 1/\theta$.

- Runs estimator (O'Brien 1987):

$$\theta_{u,m} = \mathbb{P}\left(X_{t+1} \leq u, \ldots, X_{t+m} \leq u \,\middle|\, X_t > u\right)$$

Leadbetter (1983), Hsing et al. (1988), Coles (2001)

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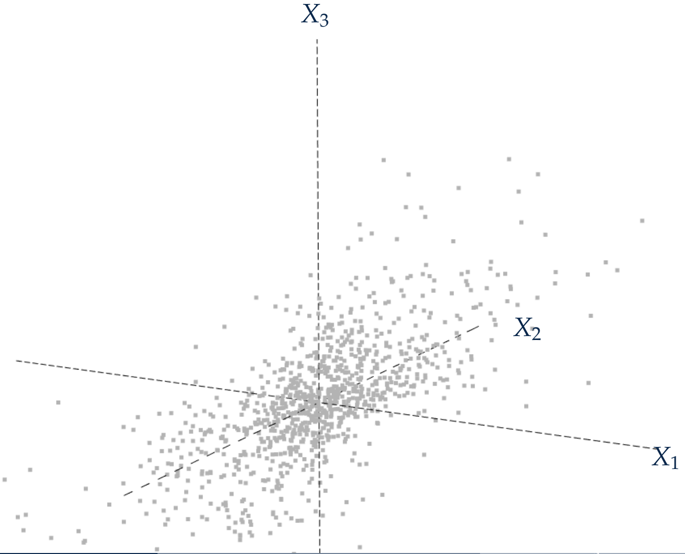
Radial-directional models in multivariate EVT

$$R := \|\boldsymbol{X}\|_2 > 0, \qquad \boldsymbol{W} := \boldsymbol{X}/R \in \mathbb{S}^{d-1}.$$



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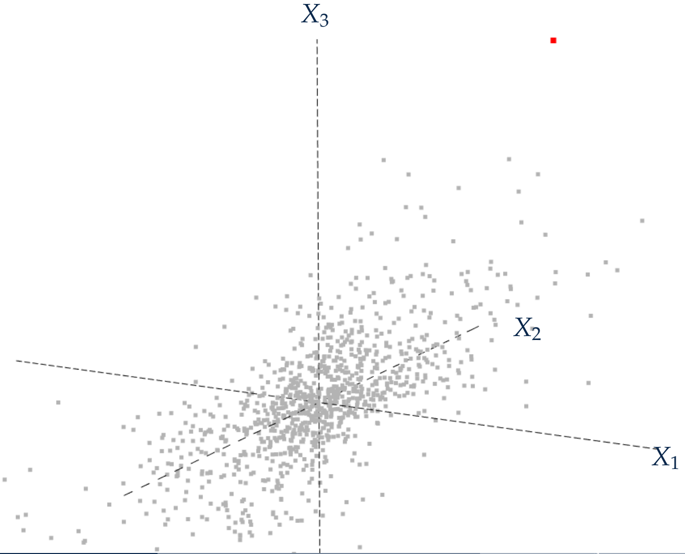

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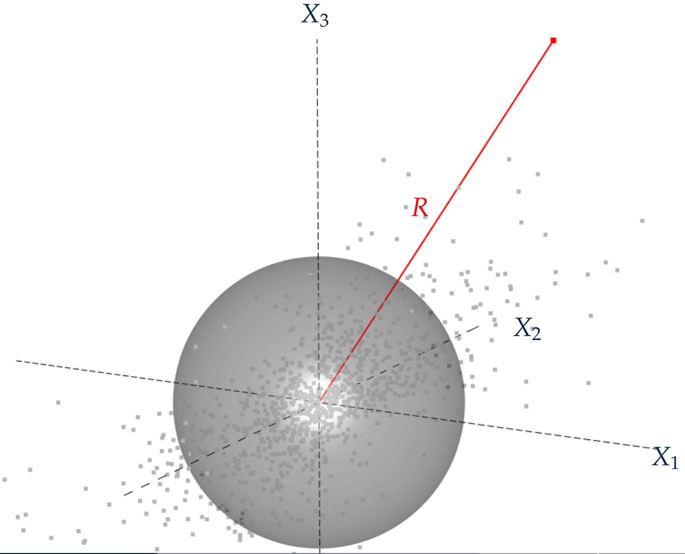

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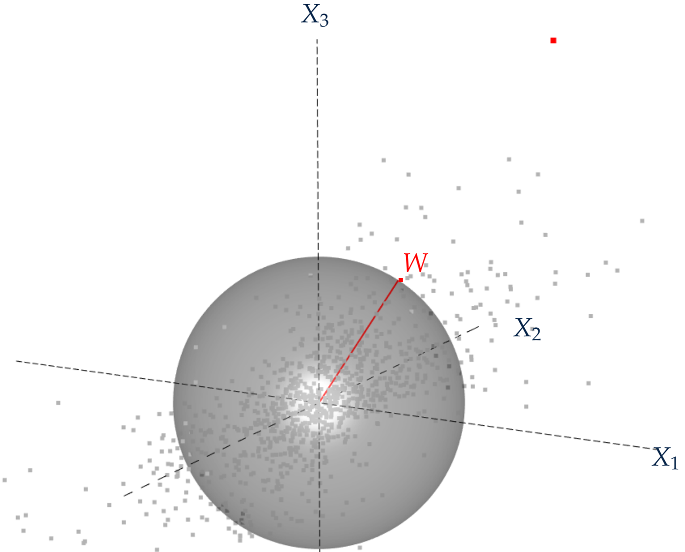

Valid multivariate thresholds and quantile sets

Definition (Multivariate threshold)

A set $\mathcal{T} \subset \mathbb{R}^d$ with the origin $\mathbf{0} \in \mathcal{T}$ defines a valid radial-directional threshold via its boundary $\partial\mathcal{T}$ if $\partial\mathcal{T}$ can be represented as a function $\partial\mathcal{T} : \mathbb{S}^{d-1} \to [0, \infty)$ and if

$$\mathcal{T} := \{r\boldsymbol{w} \in \mathbb{R}^d : 0 \leq r \leq \partial\mathcal{T}(\boldsymbol{w})\},$$

for all $\boldsymbol{w} \in \mathbb{S}^{d-1}$.

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Definition (Fixed radial quantile threshold)

A set $\mathcal{T} \subset \mathbb{R}^d$ is a quantile set of probability content $q \in (0, 1)$ if its boundary $\partial\mathcal{T}$ is defined by

$$\partial\mathcal{T}(\boldsymbol{w}) = F^{\leftarrow}_{R|\boldsymbol{W}}(q \mid \boldsymbol{w}) = \inf\{r > 0 \, : \, F_{R|\boldsymbol{W}}(r \mid \boldsymbol{w}) \geq q\},$$

where $F_{R|\boldsymbol{W}}$ and $F^{\leftarrow}_{R|\boldsymbol{W}}$ denote the distribution and quantile functions of $R \mid \boldsymbol{W}$, respectively.

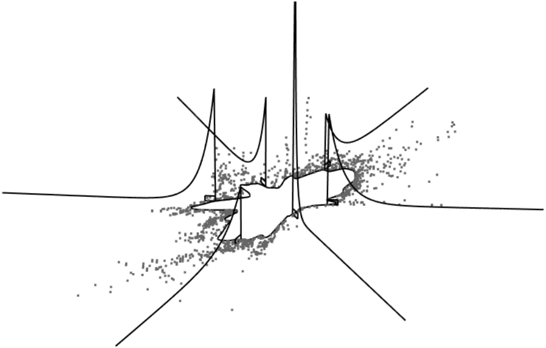
Recent radial-directional approaches

Each of these approaches (in footnote) rely on something like

$$R \mid \{R \text{ is large enough }, \boldsymbol{W} = \boldsymbol{w}\} \sim \text{ SomeUnivariateModel}$$

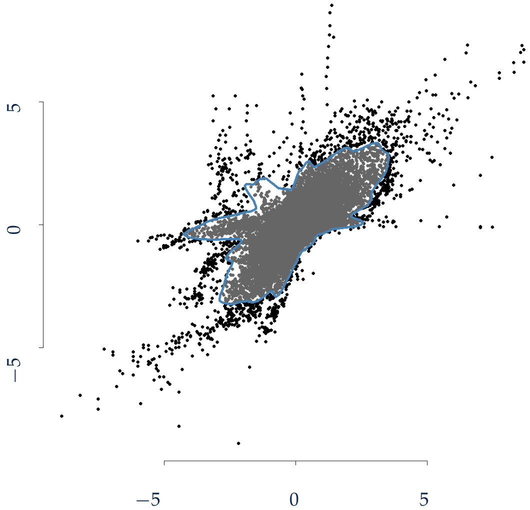
or

$$R - \partial\mathcal{T}(\boldsymbol{W}) \mid \{R > \partial\mathcal{T}(\boldsymbol{W}), \boldsymbol{W} = \boldsymbol{w}\} \sim \text{ SomeUnivariateModel}$$



Wadsworth & Campbell (2024), Simpson & Tawn (2024), Papastathopoulos et al. (2025), Murphy-Barltrop et al. (2024), De Monte et al. (2025), Murphy-Barltrop et al. (2025)

Temporal clustering of extreme events



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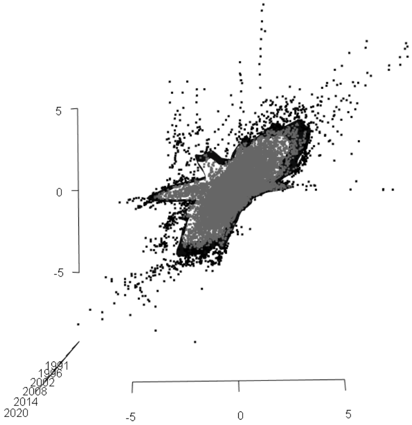
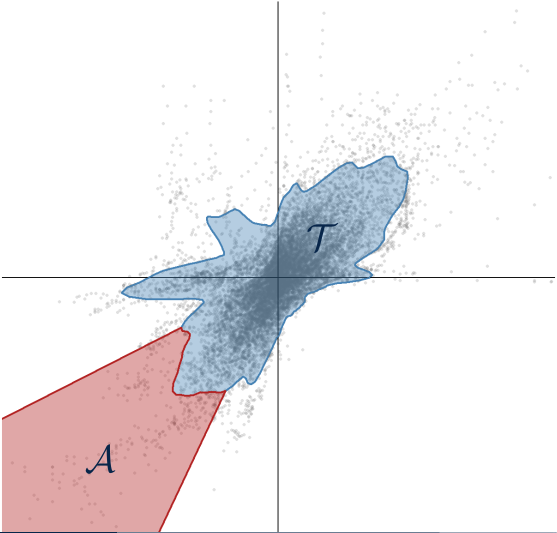


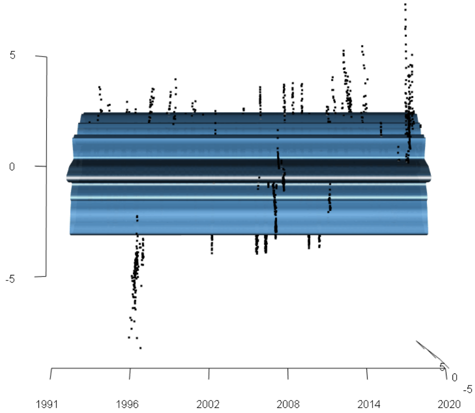
Multivariate extremal index

For some $\mathcal{A} \subseteq \mathbb{S}^{d-1}$, we define the directional extremal index

$$\Theta_{\mathcal{T},m}(\mathcal{A}) := \mathbb{P}\left[\boldsymbol{X}_{i+1} \in \mathcal{T}, \ldots, \boldsymbol{X}_{i+m} \in \mathcal{T} \mid \boldsymbol{X}_i \notin \mathcal{T}, \frac{\boldsymbol{X}_i}{\|\boldsymbol{X}_i\|} \in \mathcal{A}\right]$$



Direction-wise path of exceedances



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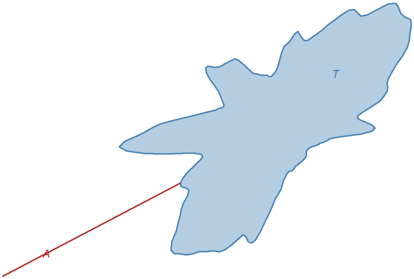

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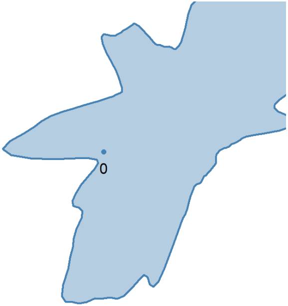
Radial-directional clusters

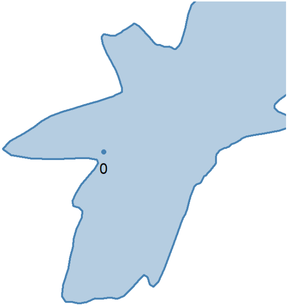
Definition

A cluster $\mathcal{C}$ of exceedances of a threshold $\mathcal{T}$ of runs length m is the sequence of all successive times between and including some $i_j, i_{j'} \in \mathcal{I}$ with $j \leq j'$ such that $(i_j - i_{j-1}) > m$, $(i_{j'+1} - i_{j'}) > m$, and $i_k - i_{k+1} \leq m$ for any $j \leq k < j'$. This cluster has size $\#\mathcal{C} = (i_{j'} - i_j + 1)$.

m = 3



m = 5



Directional extremal index

Directional mean cluster size (runs of length m):

$$
\begin{aligned}
C_{T,m}(\boldsymbol{w}) &:= \frac{1}{\theta_{T,m}(\boldsymbol{w})} \\
&= \mathbb{E}\left[\text{“Number of exceedances before } m \text{ non-exceedances”} \,\middle|\, \boldsymbol{X} \notin \mathcal{T}, \frac{\boldsymbol{X}}{\|\boldsymbol{X}\|} = \boldsymbol{w}\right]
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Aggregated mean cluster size (runs of length m):

$$C_{T,m}(\mathcal{A}) := \int_{\mathcal{A}} \frac{1}{\theta_{\mathcal{T},m}(\boldsymbol{w})} \, \mathbb{P}_{\boldsymbol{X}/\|\boldsymbol{X}\|}(\mathrm{d}\boldsymbol{w})$$

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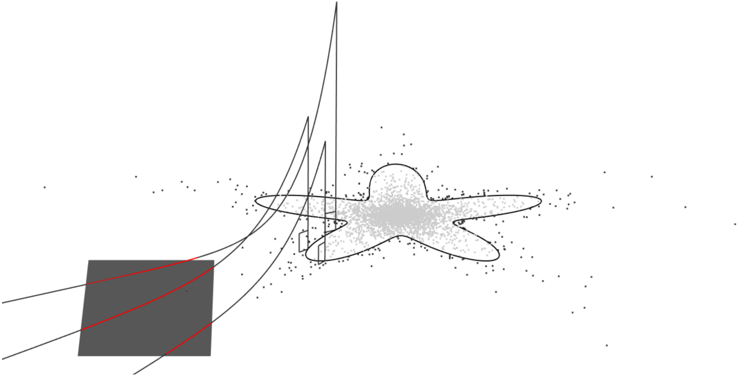
Probability estimation – A cluster's radial maxima

The probability that the radial-maximum event in a cluster lands in a risk region $\mathcal{R}$ is given by

$$\mathbb{P}[c_{\text{RM}}(\mathcal{C}) \in \mathcal{R}] = (1-q) \int_{\mathbb{S}^{d-1}} \int_{\partial\mathcal{T}(\boldsymbol{w})}^{\infty} \mathbb{1}_{r\boldsymbol{w}}[\mathcal{R}]\, \theta(\boldsymbol{w}) \,\exp\left[\frac{r - \partial\mathcal{T}(\boldsymbol{w})}{\Sigma(\boldsymbol{w})}\right] f_W(\boldsymbol{w}) \mathrm{d}\, r \,\mathrm{d}\, \boldsymbol{w}.$$



Bayesian inference pipeline

1) Standardise and learn a multivariate threshold

- Margins transformed to standard Laplace for geometric EVT.
- Infer $\mathcal{T}$ (e.g., quantile set) from data with uncertainty.
- SPDE/GMRF on $\mathbb{S}^{d-1}$ for flexible direction-dependent radial functions.

2) Learn the directional extremal index $\theta_{T,m}$

- Build binary outcomes after exceedances: $z_i = 1$ if next m observations fall in T, else 0.
- Logistic link over directions: $\theta_{T,m}(w) = \frac{1}{1+\exp\{\beta + s(w)\}}$ with $s(w)$ a cyclic/spherical spline with GMRF prior.
- Sample over posterior draws of T to propagate threshold uncertainty into $\theta_{T,m}$.

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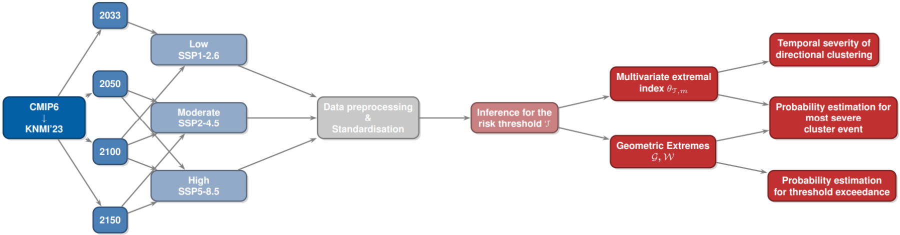

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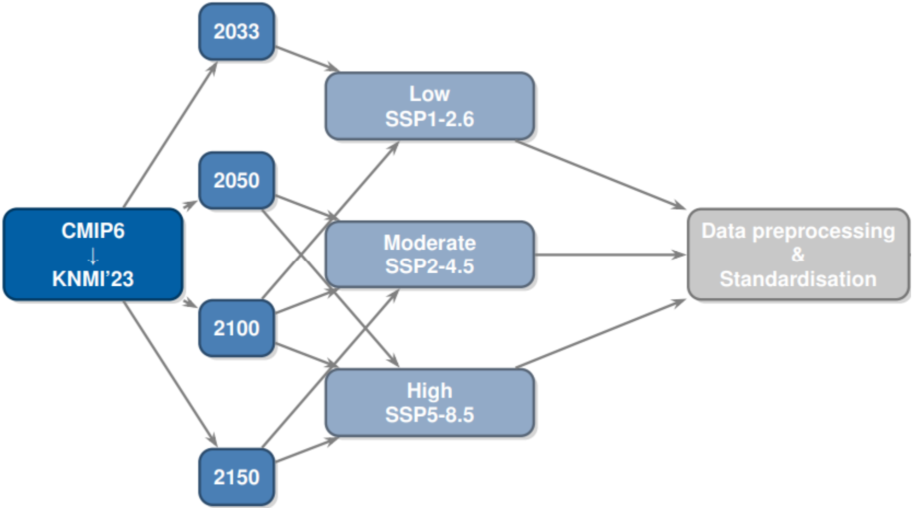
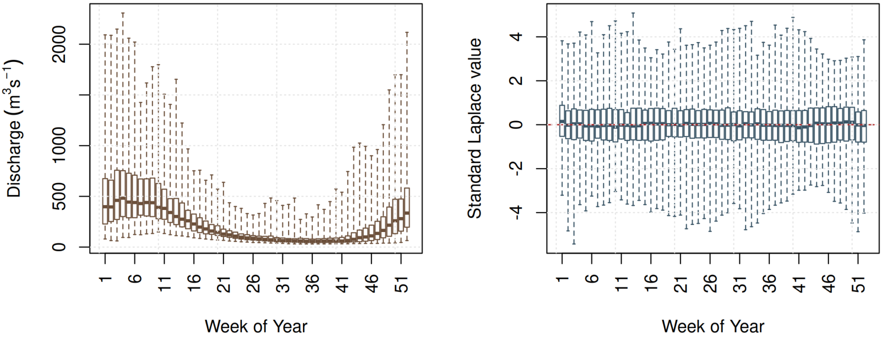
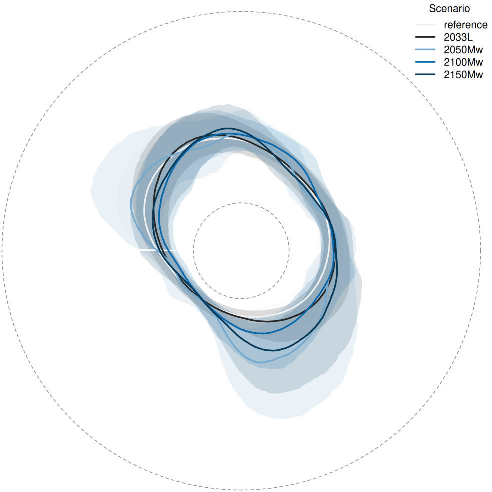


Homogenisation



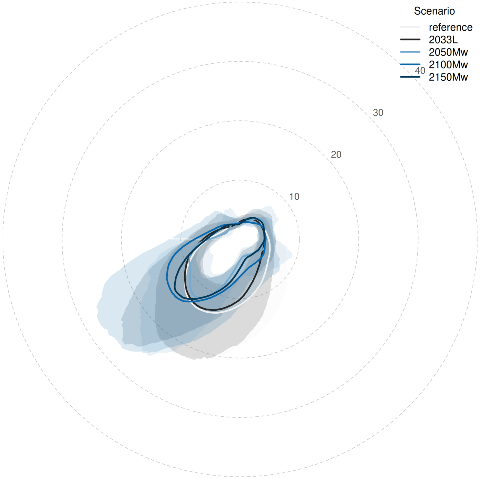
Boxplots of the yearly- and weekly-aggregated original and homogenized discharges for the Meuse and Rhine, pooling all samples across ensemble members from the 2033L scenario.

Directional extremal index



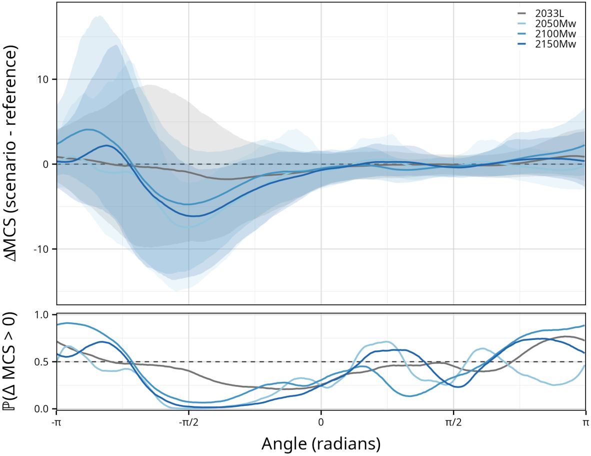
Posterior mean and 95% marginal credible intervals of the extremal index for the scenarios reference, 2033L, 2050Mw, 2100Mw and 2150Mw based on $q = 0.99$ and a runs declustering length of $m = 10$.

Directional mean cluster size



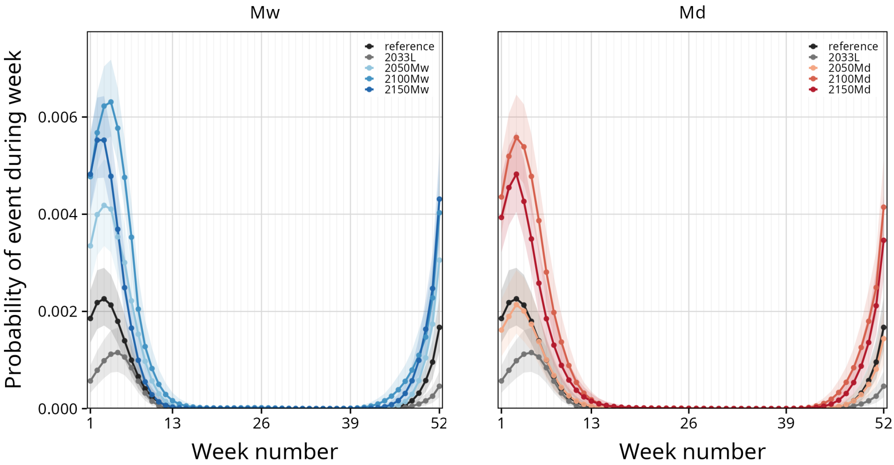
Posterior mean and 95% marginal credible intervals of the mean cluster size for the scenarios reference, 2033L, 2050Mw, 2100Mw and 2150Mw based on $q = 0.99$ and a runs declustering length of $m = 10$.

Direction-wise clustering comparison



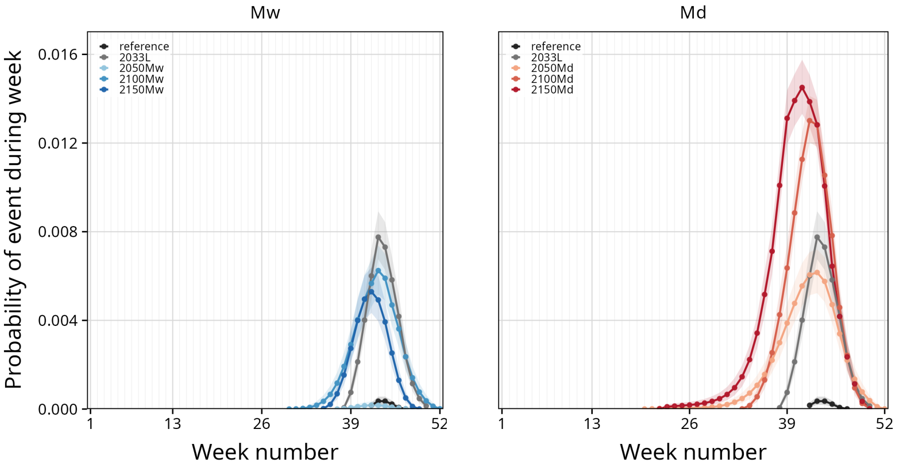
Posterior comparison of directional mean cluster size relative to the reference scenario for the Mw pathway based on $q = 0.99$ and runs declustering length $m = 10$. The upper panels show the posterior median difference $\Delta\mathrm{MCS}(\mathbf{w}) = \mathrm{MCS}_s(\mathbf{w}) - \mathrm{MCS}_{\mathrm{ref}}(\mathbf{w})$ with 95% credible intervals as a function of angular direction **w**. The lower panels show the posterior probability $\mathbb{P}(\Delta\mathrm{MCS}(\mathbf{w}) > 0)$, with the dashed line indicating probability 0.5.

Probability of high-surge events



Weekly probability estimates of Mw (left) and Md (right) scenarios across time horizons for $\mathbb{P}(Q_R \geq 12{,}000, Q_M \geq 3{,}000)$.

Probability of low-surge events



Weekly probability estimates of Mw (left) and Md (right) scenarios across time horizons for $\mathbb{P}(Q_R \leq 500, Q_M \leq 20)$.

Conclusions

- Methodological contributions:
  - Adapt runs-based declustering methods from univariate theory to form approximately independent cluster.
  - Cluster functional: radial-maximum (analogue of univariate cluster max).
  - Estimation of direction-specific mean cluster sizes $C_{T,m}(w)$.
  - Posterior predictive probabilities for cluster-level events (return regions/periods).
- Hydrological extremes near Rotterdam:
  - Little evidence against a change of clustering with respect to "new normals".
  - Most of the change in the hydrological extremes appear to be marginal.
  - Relatively clear change in the frequency of extremes of low and high discharges with climate change.

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  - Adapt runs-based declustering methods from univariate theory to form approximately independent cluster.
  - Cluster functional: radial-maximum (analogue of univariate cluster max).
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